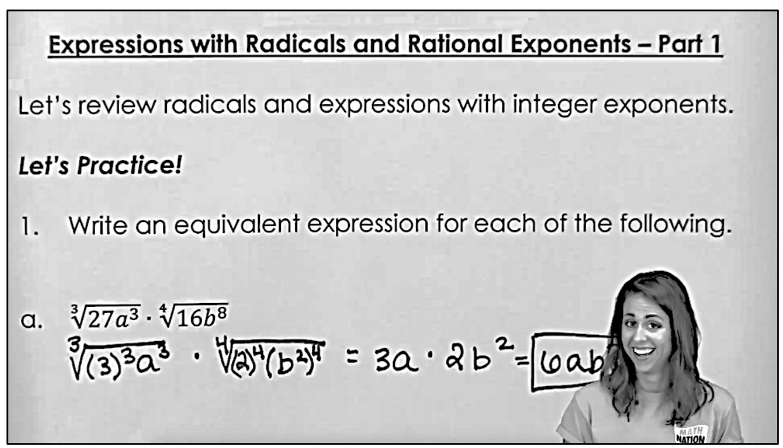
Section 8: Expressions and Equations with Radicals and Rational Exponents

Visit MathNation.com or search "Math Nation" in your phone or tablet's app store to watch the videos that go along with this workbook!



The following Mathematics Florida Standards will be covered in this section:
A-CED.1.1 - Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational, absolute, and exponential functions.
A-CED.1.4 - Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law, $V = IR$, to highlight resistance, R.*
A-REI.1.2 - Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.
F-BF.2.3 - Identify the effect on the graph of replacing $f(x)$ by $f(x)+k$, $kf(x)$, $f(kx)$, and $f(x+k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*
F-IF.2.4 - For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*
F-IF.2.5 - Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function $h(n)$ gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*
F-IF.3.7b - Graph functions expressed symbolically and show key features of the graph by hand in simple cases and using technology for more complicated cases. b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
F-IF.3.9 - Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*
N-RN.1.1 - Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define $5^{\frac{1}{3}}$ to be the cube root of 5 because we want $(5^{1/3})^{3} = 5^{(1/3)3}$, to hold, so $(5^{1/3})^{3}$ must equal 5.*
N-RN.1.2 - Rewrite expressions involving radicals and rational exponents using the properties of exponents.

Section 8: Expressions and Equations with Radicals and Rational Exponents

Section 8 – Topic 1

Expressions with Radicals and Rational Exponents – Part 1

Let's review radicals and expressions with integer exponents.

Let's Practice!

1. Write an equivalent expression for each of the following expressions.

a. $\sqrt[3]{27a^3} \cdot \sqrt[4]{16b^8}$

b. $\sqrt[4]{81x^2y^5}$

c. $\sqrt{\dfrac{4x^4}{(256y^8)^{-2}}}$

d. $\left(\dfrac{x^{-5}y^{-3}}{z^3}\right)^{-5}$

e. $\dfrac{4y^{3x-3}}{2y^{2x+4}}$

Try It!

2. Write an equivalent expression for each of the following.

a. $\sqrt[5]{-32x^{10}y^5} \cdot \sqrt[3]{8x^{21}y^6}$

b. $\sqrt[3]{\dfrac{27x^9}{(216z^6)^{-3}}}$

c. $\dfrac{6^{-1}a^2b^{-3}}{3^{-2}a^{-5}b^2}$

d. $\dfrac{a^{3b+2}\cdot a^{2b}}{a^{-2b-5}}$

Let's review rational exponents.

Let's explore why $\sqrt[3]{5} = 5^{\frac{1}{3}}$.

$\left(\sqrt[3]{5}\right)^3 = \sqrt[3]{5} \cdot \sqrt[3]{5} \cdot \sqrt[3]{5} = \sqrt[3]{125} =$ __________

$\left(5^{\frac{1}{3}}\right)^3 = 5^{\frac{1}{3}} \cdot 5^{\frac{1}{3}} \cdot 5^{\frac{1}{3}} = 5^{\left(\frac{1}{3}+\frac{1}{3}+\frac{1}{3}\right)} =$ __________

Hence, $\sqrt[3]{5} =$ ________.

Let's Practice!

3. Explain why $\sqrt[3]{6^2} = 6^{\frac{2}{3}}$.

Definition of Rational Exponents: $a^{\frac{m}{n}} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$

Section 8 – Topic 2
Expressions with Radicals and Rational Exponents – Part 2

Let's Practice!

1. Rewrite the following expressions using rational exponents.

$(6d)^{\frac{1}{4}} =$

2. Rewrite the following expressions using rational exponents.

a. $\sqrt[3]{(x+2)^4}$

b. $\sqrt[6]{y} \cdot \sqrt[3]{y} \cdot \sqrt[5]{y^2}$

c. $\sqrt[3]{\dfrac{8x^7}{(27y^5)^{-1}}}$

d. $\sqrt{\sqrt[3]{(y+1)^4}}$

Try It!

3. Rewrite the following expression using radicals.

$(10y)^{\frac{2}{3}}$

4. Rewrite the following expressions using rational exponents.

a. $\sqrt[4]{(2x-3)^3}$

b. $\sqrt[4]{\sqrt{(x-1)^5}}$

c. $\sqrt{\dfrac{4x^{11}}{(256y^8)^{-2}}}$